Exam 2 Practice Problems
Chapter 3 and 4.1-4.5

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1. Find the first and second derivative

- $x^3 - 3(x^2 + \pi^2)$
- $(x+1)^2 e^{x^3}$
- $\ln(\cos(1/x)))$
- $\cot^3(2/t)$
- $(2x+1)\sqrt{2x+1}$
- 9^{2t}
- $\frac{\sqrt{t}}{1+\sqrt{t}}$
- $\frac{1}{4}xe^{4x} - \frac{1}{16}e^{4x}$

2. Use implicit differentiation to find dy/dx and dx/dy

- $x^2y^2 = 1$
- $y^2 = \sqrt{\frac{1+x}{1-x}}$
- $x^y = \sqrt{2}$
- $ye^{\tan^{-1} x} = 2$
- $5x^{4/5} = 10y^{6/5}$

3. Word Problems (Interpreting the Derivative and Related Rates)

- The surface area S of a right, circular cylinder is related to the base radius and height by the formula

  $$S = 2\pi r^2 + 2\pi rh.$$

  Assuming height is constant, how is dS/dt related to dr/dt? (For a more difficult problem, do not assume anything is constant and relate dS/dt to both dr/dt and dh/dt).

- Show that the tangent to the curve $y = x^3$ at any point (a, a^3) meets the curve again at a point where the slope is four times the slope at (a, a^3).

- The volume of a cube is increasing at a rate of 1200 cm^3/min at the instant its edges are 20 cm long. At what rate are the lengths of the edges changing at that instance?

- The position at time $t \geq 0$ of a particle moving along a coordinate line is

  $$s = 10\cos(t + \pi/4).$$

  Find the particle's starting position, furthest distance left and right, and its velocity,speed and acceleration.

- Find the values of h, k and a that make the circle $(x-h)^2 + (y-k)^2 = a^2$ tangent to the parabola $y = x^2 + 1$ at the point (1,2) and that also makes the second derivative of the two curves equal at this point.

- A bus will hold 60 people. The number x of people per trip who use the bus is related to the fare charged (p dollars) by the law $p = (3 - \frac{x}{40})^2$. Write an expression for the total revenue per trip received by the bus company and find the maximum revenue.

4. Graph Sketching and Extreme Values (Find all critical points, inflection points, relative extrema and absolute extrema. Finish with a sketch of the graph.)

- $g(x) = \frac{x^2}{4-x^2}, \quad -2 < x \leq 1$
- $f(x) = e^{2/x}$
- $h(x) = (2 - x^2)^{3/2}$
- $m(x) = \sin x \cos x, \quad 0 \leq x \leq \pi$

5. Sketch the general shape of the graph of $y = f(x)$ given y'

- $y' = 2 + x - x^2$
- $y' = x(x-3)^2$
- $y' = 1 - \cot^2\theta, \quad 0 < \theta < \pi$
- $y' = (x^2 - 2x)(x-5)^2$

6. Use L'Hopital's Rule

- $\lim_{t\to 0} \frac{\sin t^2}{t}$
- $\lim_{x\to 0} \frac{x^2}{\ln \sec x}$
- $\lim_{x\to 1^+} x^{1/(1-x)}$
- $\lim_{x\to 0} \frac{x^2-2x}{x^2-\sin x}$
- $\lim_{x\to 0^+} \left(\frac{3x+1}{x} - \frac{1}{\sin x}\right)$
- $\lim_{x\to 0^+} \csc x - \cot x + \cos x$
- $\lim_{x\to\infty} x^{1/\ln x}$
- $\lim_{x\to\infty} \left(1+\frac{1}{x}\right)^x$
- Find a value c that makes the function

$$f(x) = \begin{cases} \frac{9x-3\sin 3x}{5x^3} & x \neq 0 \\ c & x = 0 \end{cases}$$

continuous at $x = 0$.